**COMPUTER SCIENCE DEPARTMENT**

NUMERICAL COMPUTING

**Assignment # 01**

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**Submitted to: Dr. Sajjad Ahmed Ghauri**

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**Question no.1**

Find the roots the following function using bisection method by taking suitable initial guesses.

**F(x) = ln(x)**

**Solution:**

To find roots we put some values in the equation

Let x be **0** F(0) = ln(0) => undefine

Let x be **0.5** F(0.5) = ln(0.5) => -0.6931

Let x be **1** F(1) = ln(1) => 0

Let x be **2** F(2) = ln(2) => 0.6932

Now as we can see our roots lies in the interval [0.5 , 2]

a = 0.5

b = 2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **a** | **b** | **F(a)** | **F(b)** | **x=(a+b)/2** | **F(x)** | **Condition** |
| 1 | 0.5 | 2 | -0.6931 | 0.6931 | 1.25 | 0.2231 | F, x=b |
| 2 | 0.5 | 1.25 | -0.6931 | 0.2231 | 0.875 | -0.1335 | T, x=a |
| 3 | 0.875 | 1.25 | -0.1335 | 0.2231 | 1.0625 | 0.0606 | F, x=b |
| 4 | 0.875 | 1.0625 | -0.1335 | 0.0606 | 0.9688 | -0.0317 | T, x=a |
| 5 | 0.9688 | 1.0625 | -0.0317 | 0.0606 | 1.057 | 0.0156 | F, x=b |
| 6 | 0.9688 | 1.0157 | -0.0317 | 0.0156 | 0.9923 | -0.0077 | T, x=a |
| 7 | 0.9923 | 1.0157 | -0.0077 | 0.0156 | 1.004 | 0.0039 | F, x=b |
| 8 | 0.9923 | 1.004 | -0.0077 | 0.0039 | 0.9982 | -0.0018 | T, x=a |
| 9 | 0.9982 | 1.004 | -0.0018 | 0.0039 | 1.0011 | 0.0010 | F, x=b |
| 10 | 0.9982 | 1.0011 | -0.0018 | 0.0010 | 0.9997 | -0.0003 | T, x=a |

Approximate root of the equation **ln(x)** using bisection method is 0.9997 (correct to 3dp) (after 10 iterations)

**Question no.2**

Find the roots the following function using bisection method by taking suitable initial guesses, correct to 3db.

**F(x) = x4 + x3 + x2 - 4x + 9**

**Solution:**

To find roots we put some values in the equation

Let x be **-2** F(-2) = (-2)4 + (-2)3 + (-2)2 – 4(-2) + 9 => 29

Let x be **-1** F(-1) = (-1)4 + (-1)3 + (-1)2 – 4(-1) + 9 => 14

Let x be **0** F(0) =(0)4 + (0)3 + (0)2 – 4(0) + 9 => 9

Let x be **0.5** F(0.5) = (0.5)4 + (0.5)3 + (0.5)2 – 4(0.5) + 9 => 7.4345

Let x be **1** F(1) = (1)4 + (1)3 + (1)2 – 4(1) + 9 => 8

Let x be **2** F(2) = (2)4 + (2)3 + (2)2 – 4(2) + 9 => 29

We could not find any roots intervals in the equation.

We could not apply bisection method on this equation because bisection needs two initial points ‘a’ and ‘b’ where F(a) and F(b) are opposite. It is not possible to apply bisection method on this equation this equation is a positive linear equation on x and y axis. Whereas some other method to find the error in the equation such that: Newton Raphson method, Fixed point alteration.

**Question no.3**

Find a root of equation **sin x – 2x = 0** correct to 4db. Perform six alterations.

**Solution:**

To find roots we put some values in the equation

Let x be **-1** F(-1) = sin(-1) - 2(-1) => -1.1585

Let x be **0** F(0) = sin(0) - 2(0) => 0

Let x be **0.5** F(0.5) = sin(0.5) - 2(0.5) => -0.5206 \

Now as we can see our roots lies in the interval [-1 , 0.5]

a = -1

b = 0.5

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **a** | **b** | **F(a)** | **F(b)** | **x=(a+b)/2** | **F(x)** | **Condition** |
| 1 | -1 | 0.5 | 1.1585 | -0.5206 | -0.25 | 0.2526 | T, x=a |
| 2 | -025 | 0.5 | 0.2526 | -0.5206 | 0.125 | -0.1253 | F, x=b |
| 3 | -0.25 | 0.125 | 0.2526 | -0.1253 | -0.0625 | 0.0625 | T, x=a |
| 4 | -0.0625 | 0.125 | 0.0625 | -0.1253 | 0.0313 | -0.0313 | F, x=b |
| 5 | -0.0625 | 0.313 | 0.0625 | -0.0313 | -0.0156 | 0.0156 | T, x=a |
| 6 | -0.0156 | 0.313 | 0.0156 | -0.0313 | 0.0078 | -0.0078 | F, x=b |

Approximate root of the equation **sin x – 2x = 0** using bisection method is 0.9997 (correct to 3dp) (after 6th iterations)

**Question no.4**

If we wish to compute **cos x** accuratewith five significant digits, the question is, how many terms in the expansion are to be included?

**Solution:**

Let x be **-2** F(-2) = cos (-2) => 0.99939

Let x be **-1** F(-1) = cos (-1) => 0.99984

Let x be **0** F(0) = cos (0) => 1

Let x be **0.5** F(0.5) = cos (0.5) => 0.99998

Let x be **1** F(1) = cos (1) => 0.99984

Let x be **2** F(2) = cos (2) => 0.99939

We could not find any roots intervals in the equation.

We could not apply bisection method on this equation because bisection needs two initial points ‘a’ and ‘b’ where F(a) and F(b) are opposite. It is not possible to apply bisection method on this equation this equation is a positive linear equation on x and y axis. Whereas some other method to find the error in the equation such that: Newton Raphson method, Fixed point alteration.

**Question no.5**

If the maximum number **0.22162 – 0.22586 + 4.513** are rounded, estimate the maximum absolute and relative errors. Also find the range in which the true answer lies.

**Solution:**

0.22162 – 0.22586 + 4.513

let

x1 = 0.22162 => e1 = ½\*10-6

x2 = 0.22586 => e2 = ½\*10-5

x3 = 4.513 => e3 = ½\*10-3

Z = x1 – x2 + x3

Z = 0.22162 – 0.22586 + 4.513

Z = 4.507302 =>**Actual Value**

A.E = | e1| + | e2| + |e3|

A.E = |½\*10-6| + |½\*10-5| + |½\*10-3|

A.E = ½\*10-3 (10-3 + 10-2 + 1)

A.E = ½\*10-3 (0.001 + 0.01 + 1)

A.E = ½\*10-3 (1.011)

A.E = 0.5055\*10-3

**Hence, Absolute Error is 0.5055\*10-3**

R.E = A.E / |Z|

R.E = 0.5055\*10-3 / |4.507302|

R.E = 0.1122\*10-3

**Relative Error is 0.1122\*10-3**

Z – A.E <= **Range** <= Z+A.E

4.507302 – 0.5055\*10-3 <= **Range** <= 4.507302 + 0.5055\*10-3

4.51 <= Range <= 4.51

**Its range lies between 4.5067965 and 4.5078705**